

Geometric approach to tracking and stabilization for a spherical robot actuated by internal rotors [★]

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Abstract

The paper adopts a geometric approach to stabilization and tracking of a spherical robot actuated by three internal rotors mounted on three mutually orthogonal axes inside the robot. The system is underactuated and subject to nonholonomic constraints. Initially, the equations of motion are derived through Euler-Poincaré reduction. Then two feedback control laws are synthesized: the first control law addresses orientation of the robot alone, keeping the contact point arbitrary; the second control law is a tracking law that addresses the contact position and angular velocity tracking.

Key words: Differential geometry, feedback stabilization, spherical robot.

1 Introduction

A spherical mobile robot is a spherical shell with some driving mechanism mounted inside the shell to make the shell roll. The spherical structure offers certain advantages: 1) Due to the omnidirectional feature, it can change direction more quickly and within less space compared to legged or wheeled robots. 2) It is possible to enclose any type of sensors inside the shell giving mechanical protection to the sensor.

Over the last few decades, there has been considerable research in developing control for spherical mobile robots. The classification of the control of spherical robots mainly depend upon the motion generated by the internal mechanisms. The internal mechanisms generally generate motion in two different ways: a) By changing center of mass of the total system, such as, wheeled car mounted inside the shell [14], sliders [24] or the more popular pendulum mechanism [1], [13], [22], [16]. b) Using angular momentum conservation principle such as mounting gyroscopic mechanism [27] or by the internal rotors [17], [5], [30], [29]. In this note, the spherical robot under consideration is actuated by three internal rotors and motion of the sphere is due to the conservation of angular momentum of the system.

The spherical mobile robot exhibits nonholonomic con-

straints [2] and is often underactuated, in which the number of control is less than the degrees of freedom, which makes the stabilization and tracking control challenging. These constraints do not restrict the configuration space on which the dynamics evolves, but restrict some instantaneous motions. Because of the rolling without slipping constraint the spherical shell is not allowed to slide it can move only by rolling, and the forward velocity of the shell is a function of the angular velocity of the shell. In addition, the mechanism under consideration in this paper has a continuum of equilibria; every position and orientation of the sphere is an equilibrium point. The goal of this paper is to design a nonlinear controller using the ideas of geometric techniques to stabilize a particular equilibrium configuration.

For mechanical systems with a nonholonomic constraint, one of the classical approaches to derive equations of motion is by Lagrange-D'Alembert principle. However, a modern formulation uses geometric mechanics to arrive at the equations of motion. Some advantages of this approach are, the dynamics is independent of the choice of coordinates. Further, this approach is more elegant and helps to understand the structure and the intrinsic properties of the system. In geometric modelling *symmetry* can be exploited to develop a dynamical model on a reduced space. The configuration space is identified as a Lie group and the dynamic model is derived using Lagrangian reduction. The well developed theory on geometric nonholonomic mechanics is presented in [23], [15], [9], [3], [2], [26]. By symmetry we can study the dynamics of a mechanical system on a reduced space

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and the reduced equations are in the Euler-Poincaré form. Due to nonholonomic constraints the system may or may not have full symmetry as in the case of the rigid body with gravitational field, for example, a heavy top; and Euler-Poincaré equation will depend on advection term [28]. In this paper we follow this modelling tool and derive the reduced equations of motion.

Several control laws has been applied to the spherical robot with three rotors, see for example [5], [25], [17], [18]. Spherical robot system fails to satisfy one of the necessary conditions for asymptotic stabilization by smooth time-invariant feedback given by Brockett [6]. Due to this negative result, point-to-point stabilization of position and orientation of spherical robot through time invariant state-feedback is not possible. In [28], the authors present the orientation stabilization of a Chaplygin's sphere with rotors by the control Lagrangian matching condition. In recent development, [18] have done position stabilization by expressing attitude in quaternion coordinates whereas in [25] authors have taken one step forward and discussed the position and reduced attitude stabilization and position tracking control laws respectively. In this note, an attempt is made to design a novel geometric controller for position and axis stabilization of the spherical robot. This is the best one can achieve by using smooth time-invariant state feedback. The two geometric control strategies proposed are: 1) for orientation stabilization 2) for position tracking and axis stabilization. For the orientation stabilization we have attempted to construct a potential function using a modified trace function and utilize the stability notion given in [7] to derive the close loop feedback. For position stabilization and tracking, we start with a error function and use the notion of a *transport map* from differential geometry to derive the control law for desired trajectory tracking. Further, the intermediate result of this is contact point stabilization and reduced orientation stabilization. There are some early works on this approach by [19], [8] and [20] for tracking of fully actuated systems but in our case the system under consideration is underactuated.

The paper is organized as follows: In Section 2 we present the concept, description and modelling of the spherical robot. The equation of motion is derived using Lagrangian reduction theory and is on reduced space. In section 3 we formulate the control problem for orientation stabilization and then position stabilization and tracking. We identify this stabilization as position and axis stabilization. Section 4 follows the discussion and concluding remarks on the above control strategy.

2 Description of a spherical robot

Consider a spherical mobile robot which can roll without slipping on a flat surface under a uniform gravitational field. The spherical mobile robot is an interconnected rigid body system having spherical shell and three internal rotors (actuators), as shown in figure (1). All the

three rotors are placed along three mutually orthogonal axes of the sphere-body frame. To balance the mass symmetrically, the rotor is placed on one side and a dead weight is placed on the diametrically opposite side. All the rotors and dead weights are placed such that the center of mass of the robot coincides with the geometric center of the sphere. Let the sphere body coordinate frame

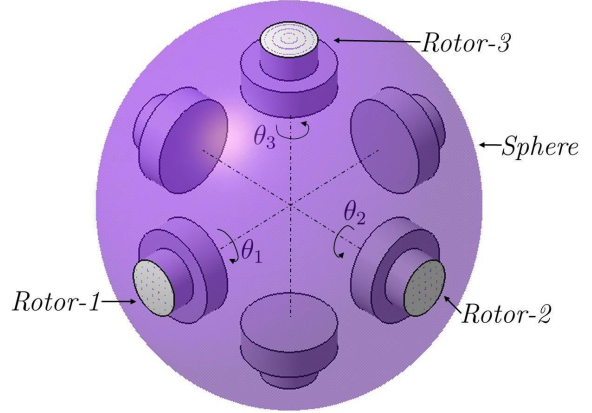


Fig. 1. Spherical robot on horizontal plane

be located with its origin at the center of the sphere. Let $\mathbf{x} \in \mathbb{R}^3$ is the position of the center of the sphere in inertial frame, and let $R_s \in SO(3)$ be the rotation matrix which maps from sphere body coordinate frame to inertial coordinate frame. The relative motion of three rotors with respect to sphere body frame is given by generalized shape coordinates $\theta_i \in S^1$, and there rotation matrices are denoted by R_i , where $i = 1, 2, 3$. Hence, the configuration space is $Q = \mathbb{R}^2 \times SO(3) \times S^1 \times S^1 \times S^1$. The following notations are adopted here:

- $(*)^I, (*)^s$:- Quantities in inertial, sphere frame respectively,
- $(\hat{e}_1, \hat{e}_2, \hat{e}_3)$ - Unit vectors in inertial frame,
- R_s, R_i - Rotation matrix of the sphere and i^{th} rotor,
- ω_s^I, ω_s^s - Angular velocity of the sphere in inertial frame and sphere frame respectively,
- $\theta_i, \dot{\theta}_i$ - Angle of rotation and angular velocity of the i^{th} rotor in i^{th} frame,
- m_s, m_i - Mass of sphere and i^{th} rotors,
- I_s - Inertia matrix of the sphere without rotors about its center of mass in sphere frame and J_i - Moment of inertia of the rotors about the three principal axes.

The Lagrangian of the system consists only of kinetic energy and is given as

$$L = \frac{1}{2} m_T \|\dot{\mathbf{x}}\|^2 + \frac{1}{2} \omega_s^T (I_s + \mathbb{J}) \omega_s + \frac{1}{2} (\dot{\Theta} \mathbb{J} \dot{\Theta} + 2 \omega_s^T \mathbb{J} \dot{\Theta}) \quad (1)$$

where $m_T = (m_s + \sum_{i=1}^3 m_i)$ is the combined mass of the sphere and rotor, $I_s = \text{diag}(I_1, I_2, I_3)$, $\mathbb{J} =$

$\text{diag}(J_1, J_2, J_3)$ and $\dot{\Theta} = (\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3)$. The rolling without slipping assumption on the robot yields a nonholonomic constraint given as

$$\dot{\mathbf{x}} - (\omega_s)^I \times r \hat{e}_3 = 0. \quad (2)$$

2.1 Dynamics of the spherical robot

Consider the configuration space Q as a smooth manifold, TQ is the velocity space called tangent bundle and a smooth distribution $\mathcal{D} \subset TQ$ defines the constraints, the set of admissible velocities. Identifying a symmetry group in a mechanical system generally allows us to reduce the number of variables and work on a smaller space. With the Lagrangian L defined in (1) and distribution $\mathcal{D} \subset TQ$ satisfying the constraint (2), let $G = SO(3) \times \mathbb{R}^3$ be a Lie group on Q with its Lie algebra $\mathfrak{g} = \mathfrak{so}(3) \times \mathbb{R}^3$, where $\mathfrak{so}(3)$ is the Lie algebra of $SO(3)$. It is seen that the Lagrangian L and distribution \mathcal{D} is invariant with respect to the subgroup $G_{\hat{e}_3}$ of G given as $G_{\hat{e}_3} = \{(R_s, b) \in G = SO(3) \times \mathbb{R}^3 | R_s^T \hat{e}_3 = \hat{e}_3\} = SO(2) \times \mathbb{R}^2$. When the Lagrangian L and the distribution \mathcal{D} are invariant under the action of the subgroup $G_{\hat{e}_3}$, the system is reduced to the space $TQ/G_{\hat{e}_3}$ and the Lagrangian is termed as the reduced Lagrangian l . Define

$$\bar{Y} = R_s^T \dot{\mathbf{x}} \quad \Gamma \triangleq R_s^T \dot{\hat{e}}_3, \quad (3)$$

where \bar{Y} is the velocity of the contact point in the sphere body frame and Γ is called an advected variable [12]. The reduced Lagrangian $l : TQ/G_{\hat{e}_3} \rightarrow \mathbb{R}$ is given as

$$l = \frac{1}{2} m_T \|\bar{Y}\|^2 + \frac{1}{2} \omega_s^T (I_s + \mathbb{J}) \omega_s + \frac{1}{2} (\dot{\Theta} \mathbb{J} \dot{\Theta} + 2 \omega_s^T \mathbb{J} \dot{\Theta})$$

and the rolling constraint is now expressed in the sphere body coordinate frame as

$$\bar{Y} = r \hat{\omega}_s^s \Gamma, \quad (4)$$

where $\bar{Y}, \Gamma \in \mathbb{R}^3$ and $\hat{\omega}_s^s = R_s^T \dot{R}_s \in \mathfrak{so}(3)$ is the (left-invariant) sphere-body angular velocity. Substituting \bar{Y} in l , the system is reduced to the quotient space $\mathcal{D}/G_{\hat{e}_3}$ given by the reduced-constraint Lagrangian l_c as

$$l_c = +\frac{1}{2} \omega_s^T (-m_T r^2 \hat{\Gamma}^T \hat{\Gamma} + I_s + \mathbb{J}) \omega_s + \frac{1}{2} (\dot{\Theta} \mathbb{J} \dot{\Theta} + 2 \omega_s^T \mathbb{J} \dot{\Theta})$$

Due to subgroup symmetry, there is an advection dynamic and differentiating (3) it is calculated as

$$\dot{\Gamma} = -\omega_s^s \times \Gamma. \quad (5)$$

With this computation, the equation of motion is given by the Euler-Poincaré equation for the group variable R_s and Euler-Lagrange equation for the shape variable

Θ , we use the intermediate theorem from [28] which is given as

$$\begin{aligned} \frac{d}{dt} \left(\frac{\partial l_c}{\partial \omega_s^s} \right) - \text{ad}^*_{\omega_s^s} \frac{\partial l_c}{\partial \omega_s^s} &= - \left(\frac{\partial l}{\partial \bar{Y}} \right) \times \dot{\Gamma} + \frac{\partial l}{\partial \Gamma} \times \Gamma, \\ \frac{d}{dt} \left(\frac{\partial l_c}{\partial \dot{\Theta}} \right) - \frac{\partial l_c}{\partial \Theta} &= u^i, \end{aligned} \quad (6)$$

where the map $\text{ad} : \mathfrak{so}(3) \times \mathfrak{so}(3) \rightarrow \mathfrak{so}(3)$ is an adjoint action of $\mathfrak{so}(3)$ on itself and $\text{ad}^* : \mathfrak{so}^*(3) \times \mathfrak{so}(3) \rightarrow \mathfrak{so}^*(3)$ is the dual of the ad -map. From (6), the dynamics is calculated as,

$$\begin{aligned} (I_s - m_T r \hat{\Gamma}^T \hat{\Gamma}) \dot{\omega}_s^s + \mathbb{J} \ddot{\Theta} - ((I_s - m_T r \hat{\Gamma}^T \hat{\Gamma}) \omega_s^s + \mathbb{J} \dot{\Theta}) \times \omega_s^s &= 0, \\ \mathbb{J} \dot{\omega}_s^s + \mathbb{J} \ddot{\Theta} &= u, \end{aligned} \quad (7)$$

where $u = [u^1 \ u^2 \ u^3]^T$ are the control torques applied to the internal rotors. Recasting the dynamic equation as,

$$\begin{aligned} M(\Gamma) \dot{\omega}_s^s &= (I_s \omega_s^s + \mathbb{J} \dot{\Theta}) \times \omega_s^s - u, \\ \mathbb{J} \dot{\omega}_s^s + \mathbb{J} \ddot{\Theta} &= u. \end{aligned} \quad (8)$$

where $M(\Gamma) = I_s - m_T r \hat{\Gamma}^T \hat{\Gamma} - \mathbb{J}$ and using the solution ω_s^s of the equation (8), we can find the curve $R_s(t)$ by solving the reconstruction equation

$$\dot{R}_s(t) = R_s(t) \hat{\omega}_s^s \text{ with } R_s(0) = R_{s0}. \quad (9)$$

Hence, equations (8) and (5), together with the reconstruction equation (9), give the complete dynamics of the spherical robot. If $u^i = 0$, it is easily seen that any configuration is an equilibrium and hence the equilibrium manifold is the whole configuration manifold Q . By expressing the control in terms of the gradient of a potential function (or error function), the equilibrium can be changed to any desired point. A similar procedure is followed in the next two sections to achieve stabilization.

Remarks on controllability

The control vector fields $f_i, i = 1, 2, 3$ are expressed as

$$f_i = \begin{bmatrix} I_s - m_T r \hat{\Gamma}^T \hat{\Gamma} & \mathbb{J} \\ \mathbb{J} & \mathbb{J} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ \hat{e}_i \end{bmatrix} = \begin{bmatrix} -(I_s - m_T r \hat{\Gamma}^T \hat{\Gamma})^{-1} \mathbb{J} \Delta^{-1} \\ \Delta^{-1} \end{bmatrix} \hat{e}_i \quad (10)$$

where $\Delta = \mathbb{J} - \mathbb{J}(I_s - m_T r \hat{\Gamma}^T \hat{\Gamma})^{-1} \mathbb{J}$. If $u = \Delta v$ is an equivalent control input where $v = (v_1, v_2, v_3)$ is a transformed control input; then the control vector fields on $SO(3) \times Q_s \times \mathbb{R}^2$ are written as

$$f_i \simeq \begin{bmatrix} -A_i(\Gamma) \\ \hat{e}_i \end{bmatrix} \quad (11)$$

where A_i is the i^{th} column of $A = (I_s - m_T r \widehat{\Gamma}^T \widehat{\Gamma})^{-1} \mathbb{J}$. From (6), which is in the Euler-Poincaré form, we see that

$$\frac{d}{dt} \left(R_s \frac{\partial l_c}{\partial \omega_s^s} \right) = 0. \quad (12)$$

Let $\frac{\partial l_c}{\partial \omega_s^s} = (I_s - m_T r \widehat{\Gamma}^T \widehat{\Gamma}) \omega_s + \mathbb{J} \dot{\Theta}$ be the angular momentum conjugate of ω_s^s , then from (12) the inertial momentum $R_s \frac{\partial l_c}{\partial \omega_s^s}$ is conserved. Suppose that the system is initially at equilibrium, then $\frac{\partial l_c}{\partial \omega_s^s} = 0$ and therefore $\omega_s = -(I_s - m_T r \widehat{\Gamma}^T \widehat{\Gamma})^{-1} \mathbb{J} \dot{\Theta} = -A(\Gamma) \dot{\Theta}$. From (4) $\dot{Y} = rA(\Gamma) \dot{\Theta} \times \Gamma$, then the control vector fields for the complete configuration $SO(3) \times Q_s \times \mathbb{R}^2$ are expressed as

$$\bar{f}_i \simeq \begin{bmatrix} -A(\Gamma) \hat{e}_i \\ \hat{e}_i \\ rA(\Gamma) \hat{e}_i \times \Gamma \end{bmatrix} \quad (13)$$

Computing iterative Lie brackets it is seen that $\{\bar{f}_1, \bar{f}_2, \bar{f}_3, [\bar{f}_1, \bar{f}_2], [\bar{f}_1, \bar{f}_3]\}$ span the tangent space of $SO(3) \times \mathbb{R}^2$ (termed as the *fiber configuration*) at any configuration. Hence, following definitions in [29], the system is *fiber* configuration accessible at any configuration.

3 Stabilization and tracking of the spherical robot

Due to the rolling constraint, the system (8) cannot achieve the desired orientation and position asymptotically with a continuous feedback law. The proposed controller in this section shows that the best one can achieve by continuous time-invariant feedback law is the contact position and the reduced attitude stabilization. And the stabilizable manifold will be $\mathbb{R}^2 \times \mathbb{S}^2$, that is, to achieve contact position and align a particular axis of the sphere in a desired direction; which can be generalize to achieve path tracking. We propose two feedback control strategies for the system (8). 1) Orientation stabilization on $SO(3)$: we use a potential function, constructed from a modified trace function to derive a continuous feedback law for attitude stabilization. 2) Position and reduced attitude stabilization on $\mathbb{R}^2 \times \mathbb{S}^2$. Here we use the notion of a transport map and the covariant derivative of a transport map to derive the velocity error term for axis stabilization.

3.1 Orientation stabilization

The control objective here is to design a feedback control law which achieves a desired orientation R_d and zero angular velocity. The rotational system dynamics described by (8) and (9) can be expressed in the standard control form with $q = (R_s, \omega_s^s)$ as

$$\dot{q} = f(q) + g(q)u \quad (14)$$

$$f = \begin{bmatrix} R_s \widehat{\omega}_s^s \\ M^{-1} (I_s \omega_s^s \times \omega_s^s + \mathbb{J} \dot{\Theta} \times \omega_s^s) \end{bmatrix}, \quad g = [g_1 \ g_2 \ g_3] = \begin{bmatrix} 0 \\ b_i \end{bmatrix}.$$

where for notational simplification we write $M(\Gamma) = M$ and b_i^s are the columns of M^{-1} . We now define a scalar valued potential function to achieve this objective and then prove the stability of the system. Subsequently, we add a damping term to get asymptotic convergence to the equilibrium point. Let $V : Q \rightarrow \mathbb{R}$ be an error function about $R_d \in SO(3)$ constructed by a modified trace function as

$$V(R_s) = \text{trace}(K_p(I_{3 \times 3} - R_d^T R_s)). \quad (15)$$

where $K_p = \text{diag}(\lambda_1, \lambda_2, \lambda_3)$ with $\lambda_1, \lambda_2, \lambda_3 > 0$ and $\lambda_1 \neq \lambda_2 \neq \lambda_3$. The modified trace function was first employ by Chillingworth et al. [10] for the purpose of feedback stabilization. Taking time derivative of V ,

$$\begin{aligned} \frac{dV}{dt} &= \text{trace}(-K_p R_d^T \dot{R}_s) = -\text{trace}(K_p R_d^T R_s R_s^T \dot{R}_s) \\ &= -\frac{1}{2} \text{trace}([\text{skew}(K_p R_d^T R_s) + \text{sym}(K_p) R_d^T R_s](R_s^T \dot{R}_s)) \\ &= -\frac{1}{2} \text{trace}(\text{skew}(K_p R_d^T R_s)(R_s^T \dot{R}_s)) \end{aligned}$$

from the equality $\text{trace}(\widehat{xy}) = -2x \cdot y$, where $\widehat{\cdot} : \mathbb{R}^3 \rightarrow \mathfrak{so}(3)$ is a *hap* map and $(\cdot)^\vee : \mathfrak{so}(3) \rightarrow \mathbb{R}^3$ is a *brave* map (inverse of hat map), it follows

$$\begin{aligned} \dot{V} &= \text{skew}(K_p R_d^T R_s)^\vee \cdot (R_s^T \dot{R}_s)^\vee = \text{skew}(K_p R_d^T R_s)^\vee \cdot \omega_s^s \\ &= (\lambda_1 R_s^T \hat{e}_1 \times R_d^T \hat{e}_1 + \lambda_2 R_s^T \hat{e}_2 \times R_d^T \hat{e}_2 + \lambda_3 R_s^T \hat{e}_3 \times R_d^T \hat{e}_3) \cdot \omega_s^s, \\ &= \left(\sum_{i=1}^3 \lambda_i R_s^T \hat{e}_i \times R_d^T \hat{e}_i \right) \cdot \omega_s^s = dV \cdot \omega_s^s. \end{aligned}$$

Theorem 1 Under the feedback torque $u(R_s) = dV(R_s)$ the closed loop system (14) is Lyapunov stable about $(R_d, 0)$.

Proof: Define the function $H : Q \rightarrow \mathbb{R}$

$$H(R_s, \omega_s^s) = \frac{1}{2} \omega_s^s \cdot M \omega_s^s + V(R_s) \quad (16)$$

The point $(R_d, 0)$ is an isolated equilibrium point of the system (14). Note that V is an error function about R_d and since $M^{-1} > 0$, therefore it follows that the function H is locally positive definite around $(R_d, 0)$. It follows that

$$\begin{aligned} \frac{dH}{dt} &= \omega_s^s \cdot M \dot{\omega}_s^s + \frac{1}{2} (\omega_s^s)^T \frac{dM}{dt} \omega_s^s + \dot{V}, \\ &= \omega_s^s \cdot ((I_s \omega_s^s + \mathbb{J} \dot{\Theta}) \times \omega_s^s - u) + \left(\sum_{i=1}^3 \lambda_i R_s^T \hat{e}_i \times R_d^T \hat{e}_i \right) \cdot \omega_s^s, \end{aligned}$$

$$= \omega_s^s \cdot \left(-u + \sum_{i=1}^3 \lambda_i R_s^T \hat{e}_i \times R_d^T \hat{e}_i \right) = 0. \quad (17)$$

Thus, H is a Lyapunov function about $(R_d, 0)$ and therefore $(R_d, 0)$ is stable in the sense of Lyapunov for system (14). ■

The next step is to introduce damping or the dissipative term u_{diss} to achieve asymptotic stability. Introducing damping to the control by defining $u = dV + u_{diss}$ where $u_{diss} = [u_{diss}^1 \ u_{diss}^2 \ u_{diss}^3]^T$. Then the closed loop control system becomes

$$\dot{q} = F_{cl}(q) + g(q)u_{diss} \quad (18)$$

$$\text{where } \begin{bmatrix} R_s \hat{\omega}_s^s \\ M^{-1} (I_s \omega_s^s \times \omega_s^s + \mathbb{J} \dot{\Theta} \times \omega_s^s + (\sum_{i=1}^3 \lambda_i R_s^T \hat{e}_i \times R_d^T \hat{e}_i)) \end{bmatrix},$$

$$g = [g_1 \ g_2 \ g_3] \quad g_i = \begin{bmatrix} [0]_{3 \times 3} \\ -b_3 \end{bmatrix}.$$

Lemma 1: The control system (18) is locally controllable on $SO(3) \times \mathbb{R}^3$.

Proof: The proof is given in Appendix 3. ■

We now prove asymptotic stability of our system about the desired equilibrium point. To prove this we use the stability result stated in [[7], theorem 1].

Theorem 2 Consider the system (18) with input torque u_{diss} . Let H be described in (16). If $\mathcal{L}_{F_{cl}} H = 0$ and $u_{diss} = -\mathcal{L}_g H$ is the dissipative input, then $(R_d, 0)$ is local asymptotically stable.

Proof: Consider the Lyapunov function H as defined in (16), Computing the rate of H we get

$$\frac{dH}{dt} = \frac{\partial H}{\partial q} \dot{q} = \mathcal{L}_{F_{cl}} H + \mathcal{L}_g H u_{diss},$$

From (17), we see that, $\mathcal{L}_{F_{cl}} H = 0$ which implies

$$\dot{H} = \mathcal{L}_g H u_{diss}. \quad (19)$$

Defining $u_{diss} = -\mathcal{L}_g H$ yield $\dot{H} = -(\mathcal{L}_g H)^2$. We know that $u_{diss} = [u_{diss}^1 \ u_{diss}^2 \ u_{diss}^3]^T$ then calculating u_{diss}^i as

$$u_{diss}^i = -\left(\frac{\partial H}{\partial q}\right)^T g_i = -\left[\left(\frac{\partial H}{\partial R_s}\right)^T \left(\frac{\partial H}{\partial \omega_s^s}\right)^T\right] \begin{bmatrix} 0 \\ -b_i \end{bmatrix} = M \omega_s^s \cdot b_i$$

where $i = 1, 2, 3$. From this the dissipative control is calculated as

$$u_{diss} = -\mathcal{L}_g H = -[\mathcal{L}_{g_1} H \ \mathcal{L}_{g_2} H \ \mathcal{L}_{g_3} H]^T$$

$$= M \omega_s^s \cdot [b_1 \ b_2 \ b_3]^T = M \omega_s^s \cdot (M^{-1})^T = K_v \omega_s^s$$

where $M = M^T$ is a symmetric positive-definite matrix and K_v is a positive constant. Substituting the value of $\mathcal{L}_g H$ in (19) we get

$$\dot{H} = -(K_v \omega_s^s)^2 \leq 0.$$

Since, system is locally controllable from Lemma 1 and \dot{H} is negative semidefinite we conclude from the theorem 1 of [7] that the point $(R_d, 0)$ is local asymptotically stable. ■

To check exponential stability, we compute the second variation of H . If the second variation is positive definite about the equilibrium point we say that the equilibrium is exponentially stable. From (16) $H = T + V$ where $T = (1/2)\omega_s^s \cdot M \omega_s^s$ and for T being a kinetic energy, yields $\partial^2 T(q) > 0$ for all q . The second variation of the error functions V is calculated as follows; let $\hat{\eta} = R_s^T \delta R_s$ be an element of $\mathfrak{so}(3)$, then

$$\begin{aligned} \delta V(R_s) &= \delta(\text{trace}(K_p(I_{3 \times 3} - R_s R_d^T))) \\ &= \text{trace}(-K_p \delta R_s R_d^T) = \text{trace}(K_p \delta R_s R_s^T R_s R_d^T), \\ &= \text{trace}(K_p R_s^T \delta R_s R_s R_d^T) = \text{trace}(\hat{\eta} K_p R_s R_d^T), \\ \delta^2 V(R_s) &= \delta \text{trace}(\hat{\eta} K_p R_s R_d^T) = \text{trace}(-\hat{\eta} K_p \delta R_s R_d^T) \\ &= \text{trace}(-\hat{\eta} K_p R_s^T \delta R_s R_s R_d^T) = \text{trace}(-\hat{\eta} K_p \hat{\eta} R_s R_d^T), \\ &= \langle \hat{\eta}, K_p R_s R_d^T \hat{\eta} \rangle \quad \forall \hat{\eta} \neq 0. \end{aligned} \quad (20)$$

At R_d we have $\partial^2 V > 0$ and $\partial^2 H(q_0) = \partial^2 T(q_0) + \partial^2 V(q_0) > 0$, where $q_0 = R_d$. Since, the second variation of H is positive definite at equilibrium one can conclude the system achieves the desired orientation exponentially.

Simulation

We choose the model parameters as:

$$\begin{aligned} m_s &= 1 \text{ kg}; \ m_1 = m_3 = 0.672 \text{ kg}; \\ r_s &= 0.176 \text{ m}; \ r_1 = r_2 = r_3 = 0.02 \text{ m}; \\ \mathbb{J} &= J_i = \text{diag}(0.672, 0.672, 0.672) \text{ kg} \cdot \text{cm}^2; \\ I_s &= \text{diag}(0.0153, 0.0153, 0.0153) \text{ kg} \cdot \text{m}^2. \end{aligned}$$

and control parameters are $K_p = \text{diag}(10, 8, 4)$ and $K_v = 12$. Choosing rotation matrix R_s given as $R_s = \exp(\alpha \hat{e}_1) \exp(\beta \hat{e}_3) \exp(\gamma \hat{e}_1)$, the simulations are carried out for the attitude stabilization by three rotors with the following control law:

$$u = -\left(\sum_{i=1}^3 \lambda_i R_s^T \hat{e}_i \times R_d^T \hat{e}_i\right) + K_v \omega_s^s.$$

The initial momentum $\mu = (7, 3, 1)$ gives the initial angular velocity as $\omega_s^s(0) = (12.5, 7, 1)$. Keeping the desired orientation as $R_d = R_x(\pi/9) R_y(\pi/18) R_z(\pi/3)$, then Fig. (2(a)) and (2(b)) shows the angular velocity of the sphere and the error norm of R_s , indicating asymptotic convergence to the desired value.

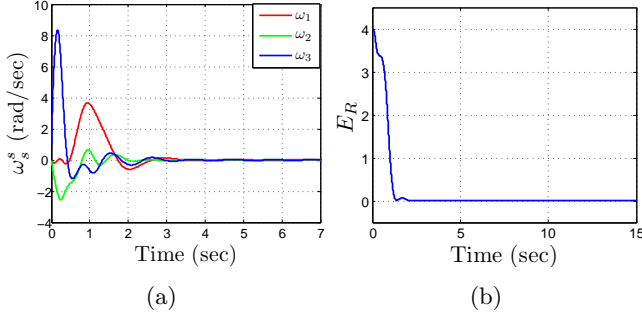


Fig. 2. (a) Angular velocity of spherical robot. (b) Error norm of orientation

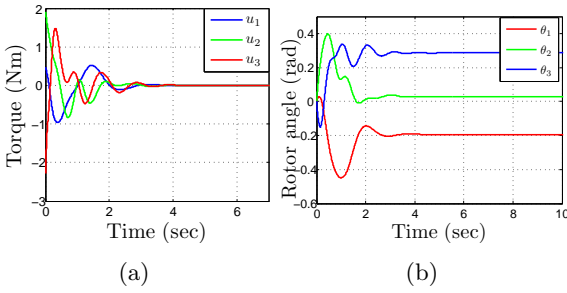


Fig. 3. (a) Torque to the internal rotors. (b) Angular position of the rotors.

The error norm is calculated as

$$E_R = (3 - \text{trace}(R_d^T R_s))^{1/2}.$$

Figure (3(a)) shows the torque applied at the internal rotors. The angular velocity of the three rotors is shown in Figure (3(b)) converges to the initial momentum.

3.2 Contact point tracking and stabilization: A geometric approach

In this section we derive a control law based on a configuration error function which ensures contact position tracking by tracking the angular velocity. The control objective is to design a control law which aligns ω_s^s to a desired angular velocity and stabilizes/tracks the contact position on the XY-plane asymptotically. Suppose $\dot{\mathbf{x}}_d = R_d \omega_d^s \times r \hat{e}_3$ is the desired contact point velocity, where R_d is a desired orientation. Note that given the nonholonomic constraint, $\dot{\mathbf{x}}_d$ gets determined by ω_s^s which eventually determine \mathbf{x}_d . Let V_1 be a potential function given by

$$V_1 = \frac{1}{2} \|\mathbf{x} - \mathbf{x}_d\|^2. \quad (21)$$

Taking the time derivative of V along the system's trajectory,

$$\begin{aligned} \dot{V}_1 &= (\mathbf{x} - \mathbf{x}_d) \cdot (\dot{\mathbf{x}} - \dot{\mathbf{x}}_d) \\ &= (\mathbf{x} - \mathbf{x}_d) \cdot [(R_s \omega_s^s \times r \hat{e}_3) - (R_d \omega_d^s \times r \hat{e}_3)], \quad (22) \\ &= -r \hat{e}_3 \times (\mathbf{x} - \mathbf{x}_d) \cdot R_s \omega_s^s - R_d \omega_d^s, \\ &= -r \hat{e}_3 \times (\mathbf{x} - \mathbf{x}_d) \cdot R_s (\omega_s^s - R_s^T R_d \omega_d^s), \\ &= -r R_s^T [\hat{e}_3 \times (\mathbf{x} - \mathbf{x}_d)] \cdot (\omega_s^s - R_s^T R_d \omega_d^s), \end{aligned}$$

Set $R_e = R_d^T R_s$ and define the error in angular velocity as $e_\omega \triangleq \omega_s^s - R_e^T \omega_d^s$. The proportional-derivative (PD) control term is given as

$$f_{PD} = k_p r R_s^T [\hat{e}_3 \times (\mathbf{x} - \mathbf{x}_d)] - k_d e_\omega, \quad (23)$$

where k_p and k_d are positive definite matrices. In tracking problems, the time-dependent state feedback has a control interpretation as feedback plus feedforward control in the sense that the time dependence is introduced by the desired trajectory ω_d^s and its derivative. Hence, we compute the feedforward (FF) control term which tracks the desired velocity and add the proportional-derivative (PD) term to stabilize/tracks the XY-position asymptotically. The velocity error has geometric interpretation since $\hat{\omega}$ is Lie algebraic element. Since, ω_s^s and ω_d^s are the two velocities taking values in different tangent spaces, to define the error velocity we need to compare tangent vectors in the same tangent space. This can be achieved by the transport map τ as defined in [[21], §11]. If $\dot{R}_s \in T_{R_s} SO(3)$ and $\dot{R}_d \in T_{R_d} SO(3)$ are the two vectors at the points R_s and R_d respectively, then a right transport map $\tau(R_s, R_d)$ transforms \dot{R}_d into a vector at $T_{R_s} SO(3)$ and the error is expressed as

$$\begin{aligned} \dot{R}_s - \tau(R_s, R_d)(\dot{R}_d) &= \dot{R}_s - \dot{R}_d(R_d^T R_s), \\ &= R_s R_s^T \dot{R}_s - (R_s R_s^T)(R_d R_d^T) \dot{R}_d (R_d^T R_s), \\ &= R_s \hat{\omega}_s^s - R_s (R_s^T R_d) \hat{\omega}_d^s (R_d^T R_s), \\ &= R_s [\hat{\omega}_s^s - (R_s^T R_d \omega_d^s)^\wedge] \\ &= R_s [\hat{\omega}_s^s - (R_e^T \omega_d^s)^\wedge] = R_s \hat{e}_\omega \end{aligned}$$

Now, the feedforward control term is calculated by taking the covariant derivative of the transport map along ω_s^s . Associated with the Riemannian manifold is the notion of the affine connection ∇ that defines the covariant derivative. For details on Riemannian manifolds and affine differential geometry one can refer to [11], [4], [2]. For a given affine connection ∇ , two vector fields X_ξ, X_η with $\xi, \eta \in \mathfrak{g}$ its Lie algebra, the covariant derivative is defined as $\nabla_{X_\xi} X_\eta$. If X_ξ, X_η are left-invariant vector fields on Q , then the covariant derivative is

$$\nabla_{X_\xi} X_\eta = \frac{d}{dt} X_\eta + \frac{\mathfrak{g}}{\nabla} \xi \eta,$$

where $\overset{\mathfrak{g}}{\nabla}_\xi \eta$ is a bilinear map defined in [21] as

$$\overset{\mathfrak{g}}{\nabla}_\xi \eta = \left(\frac{1}{2} M^{-1} (\xi \times M \eta) + \frac{1}{2} M^{-1} (\eta \times M \xi) \right). \quad (24)$$

In our case $X_\xi = \dot{R}_s$, $X_\eta = \tau(R_s, R_d) \dot{R}_d$ and $\mathfrak{g} = \mathfrak{so}(3)$ with $\xi = \widehat{\omega}_s^s$ and $\eta = (R_d^T \omega_d^s)^\wedge$. With this the f_{FF} is calculated as

$$\begin{aligned} f_{FF} &= M \left(\nabla_{\dot{R}_s} \tau(R_s, R_d) \dot{R}_d \right), \\ &= M \left(\frac{d}{dt} R_e^T \omega_d^s + \overset{\mathfrak{so}(3)}{\nabla} \widehat{\omega}_s^s (R_e^T \omega_d^s)^\wedge \right) \\ &= M \left(\frac{dR_e^T}{dt} \omega_d^s + R_e^T \frac{d\omega_d^s}{dt} + \overset{\mathfrak{so}(3)}{\nabla} \widehat{\omega}_s^s (R_e^T \omega_d^s)^\wedge \right) \\ &= M \left((\omega_s^s \times R_e^T \omega_d^s) + R_e^T \dot{\omega}_d^s + \overset{\mathfrak{so}(3)}{\nabla} \widehat{\omega}_s^s (R_e^T \omega_d^s)^\wedge \right). \end{aligned}$$

From (24) calculating the bilinear map and therefore

$$\begin{aligned} f_{FF} &= M \left(\frac{1}{2} M^{-1} (\omega_s \times M R_e^T \omega_d^s) - \frac{1}{2} M^{-1} (M \omega_s \times R_e^T \omega_d^s) \right) \\ &\quad + M (\omega_s^s \times R_e^T \omega_d^s) + M R_e^T \dot{\omega}_d^s. \end{aligned} \quad (25)$$

With (23) and (25), the nonlinear controller is given as

$$u = f_{PD} + f_{FF}. \quad (26)$$

Theorem 3 *The closed loop system (8) with control input (26) given by*

$$\dot{\omega}_s^s = M^{-1} (I_s \omega_s^s + \mathbb{J} \dot{\Theta}) \times \omega_s^s - M^{-1} (f_{PD} + f_{FF}),$$

is local asymptotically stable at $(\mathbf{x}_d, R_e^T \omega_d^s)$.

Proof: Define a candidate error function

$$H(\mathbf{x}, \omega_s^s) = V_1 + \frac{1}{2} \|e_\omega\|_M^2 = V_1 + \frac{1}{2} \mathbb{G}(I)(e_\omega, e_\omega), \quad (27)$$

where $\mathbb{G}(I) = M$ is the Riemannian metric on Q . The time derivative of the Lyapunov function is

$$\begin{aligned} \frac{d}{dt} H(\mathbf{x}, \omega_s^s) &= \frac{d}{dt} V_1 + \mathbb{G}(I)(e_\omega, \nabla_{\omega_s^s} e_\omega), \\ &= \dot{V}_1 + \mathbb{G}(I)(e_\omega, \nabla_{\omega_s^s} \omega_s^s - \nabla_{\omega_s^s} R_e^T \omega_d^s), \\ &= \dot{V}_1 + \langle e_\omega, M \left(\frac{d}{dt} \omega_s^s + \overset{\mathfrak{so}(3)}{\nabla} \widehat{\omega}_s^s \widehat{\omega}_s^s \right) \rangle - \langle e_\omega, M \nabla_{\omega_s^s} R_e^T \omega_d^s \rangle, \\ &= \dot{V}_1 + \langle e_\omega, M \left(\frac{d}{dt} \omega_s^s - M^{-1} (\omega_s^s \times (I_s \omega_s^s + \sum_{i=1}^3 J_i e_i \dot{\theta}_i)) \right) \rangle \\ &\quad - \langle e_\omega, M \nabla_{\omega_s^s} R_e^T \omega_d^s \rangle, \\ &= \dot{V}_1 + \langle e_\omega, u \rangle - \langle e_\omega, M \nabla_{\omega_s^s} R_e^T \omega_d^s \rangle, \end{aligned}$$

$$\begin{aligned} &= -r R_s^T [\hat{e}_3 \times (\mathbf{x} - \mathbf{x}_d)] \cdot e_\omega + \langle e_\omega, f_{PD} + f_{FF} \rangle - \langle e_\omega, f_{FF} \rangle, \\ &= -k_d e_\omega \cdot e_\omega \leq 0. \end{aligned} \quad (28)$$

Let $\Omega_c = \{(\mathbf{x}, \omega_s^s) | H(\mathbf{x}, \omega_s^s) \leq c\}$, where $c > 0$ and since $\dot{H}(\mathbf{x}, \omega_s^s) \leq 0$ all the trajectories are bounded and contained within Ω_c . Define N to be the set of all points of Ω_c satisfying $\dot{H} = 0$. From (28), we have

$$N = \{(\mathbf{x}, \omega_s^s) \in \Omega_c | e_\omega = 0\}.$$

As $e_\omega = 0$ implies $\omega_s^s = R_e^T \omega_d^s$ which yields the dynamics, $\dot{\mathbf{x}} = \dot{\mathbf{x}}_d$ and $\dot{\omega}_s^s = -r M^{-1} R_s^T [\hat{e}_3 \times (\mathbf{x} - \mathbf{x}_d)] + \frac{d}{dt} (R_e^T \omega_d^s)$. Since the robot rolls on a horizontal plane, at any point $(\mathbf{x} - \mathbf{x}_d) \neq \hat{e}_3$. So the only possibility of $\dot{e}_\omega = 0$ to happen is when $x = x_d$. Hence, the largest invariant set will be the set $N_1 = \{(\mathbf{x}, e_\omega) | \omega_s^s = R_e^T \omega_d^s, \mathbf{x} = \mathbf{x}_d\}$ in Ω_c . And from LaSalle's invariance principle, the trajectories in Ω_c converge to N_1 as $t \rightarrow \infty$, that is, to the equilibrium $(\mathbf{x}, \omega_s^s) = (\mathbf{x}_d, R_e^T \omega_d^s)$. ■

Position and reduced attitude stabilization

For position stabilization there are two cases: 1) when $x_d = 0$ & $R_d \omega_d^s = 0 \Rightarrow \omega_d^s = 0$; and 2) when $x_d = 0$ & $R_d \omega_d^s = \alpha \hat{e}_3$, where α is any scalar. Case 1 is immediate, when $\omega_d^s = 0$ the control law $u = f_{PD}$ and the robot converges to the origin asymptotically. In case 2 when $R_d \omega_d^s = \alpha \hat{e}_3$ which implies that the robot final contact position is the origin and the angular velocity is about z -axis. The control law in this case is expressed as

$$\begin{aligned} u &= k_p r R_s^T (\hat{e}_3 \times \mathbf{x}) - k_d (\omega_s^s - \alpha R_s^T \hat{e}_3) + (\omega_s^s \times \alpha M R_s^T \hat{e}_3) \\ &\quad + M (\omega_s^s \times \alpha R_s^T \hat{e}_3). \end{aligned}$$

The first term in the control law is responsible for the contact position stabilization and remaining terms will orient the sphere such that the angular velocity tracks \hat{e}_3 . Such is a case of reduced attitude stabilization where stabilizing R_s upto a rotation about \hat{e}_3 is equivalent to stabilizing the angular velocity direction of the axis $R_s^T \hat{e}_3$ [8]. Thus, we can restate the attitude as $R_s \in \mathbb{S}^2$ and conclude that the control law (26) gives the contact point and reduced attitude stabilization in terms of the points in $\mathbb{R}^2 \times \mathbb{S}^2$.

Simulation

We take the model parameters as in section (3.1) with initial orientation $R_{s_0} = \exp(\frac{\pi}{6} \hat{e}_1)$ and starting point on the horizontal plane as $(x_0, y_0) = (4, 2)$ units. Setting the desired orientation $R_d = \exp(\frac{\pi}{4} \hat{e}_3)$ and $\omega_d^s = \hat{e}_3$ which satisfy $R_d^T \omega_d^s = \hat{e}_3$, Fig. (4(a)) and Fig. (5(a)) shows that as the angular velocity achieve ω_d^s asymptotically, the sphere attains the desired Line-of-sight that is $R_s^T \hat{e}_3 = (\Gamma_1, \Gamma_2, \Gamma_3)$ converges to $\hat{e}_3 = (0, 0, 1)$. The

initial oscillations in $R_s^T \hat{e}_3$ plot are due to the sphere rotating in the spiral type motion on plane (Fig. (6)) and then asymptotic converges to $(0, 0, 1)$. The position on xy plane is illustrated in Fig. (4(b)).

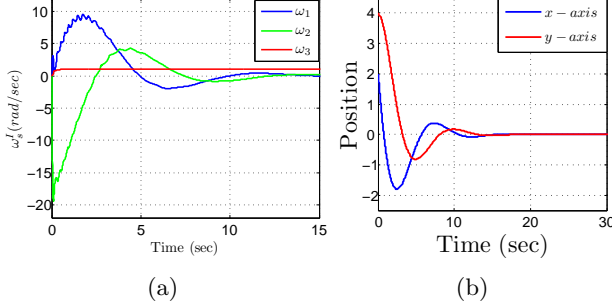


Fig. 4. a) Angular velocity of spherical robot. b) Position on the plane.

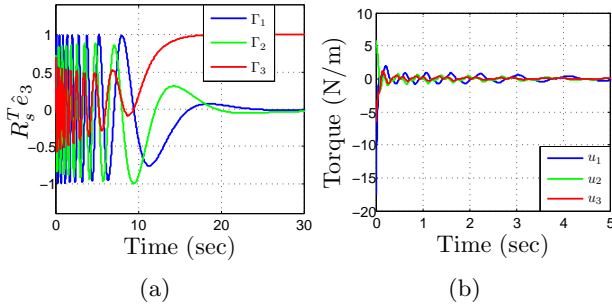


Fig. 5. Γ plot and torques on rotors.

To illustrate contact point trajectory tracking, we choose \mathbf{x}_d to track line and circle, as shown in Fig. (7), (8) and (9). Keeping $\omega_d^s = 0$, then Fig. (7)(a) and (b) shows the angular velocity of the sphere and the phase plane of position. Setting $\mathbf{x}_d = (r_s \sin(t), r_s \cos(t))$ which yields $\omega_d^s = (-\sin(t), -\cos(t))$. Fig. (8) shows the spherical robot follows the desired circular trajectory. Setting $\omega_d^s = (0.2, 0.3)$ and $\mathbf{x}_d = (0.2t + 0.4, 0.3t + 0.6)$ then the sphere will rotate at constant speed shown in Fig. (9) and follows the line given by \mathbf{x}_d .

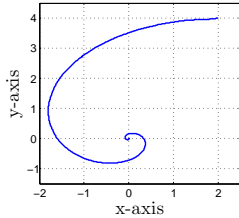


Fig. 6. Phase plane of xy position.

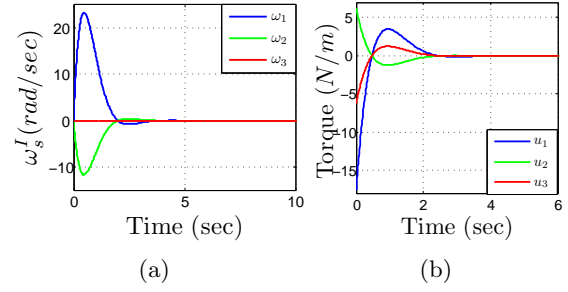


Fig. 7. Angular velocity and torques on rotors.

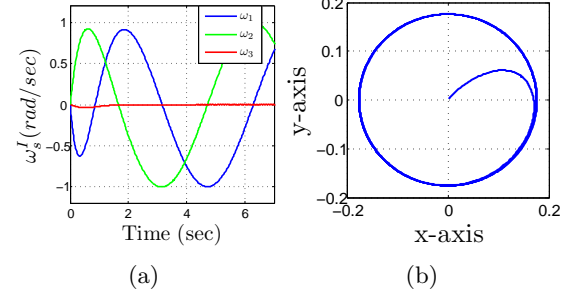


Fig. 8. Angular velocity and phase plane of xy position tracks the circular trajectory.

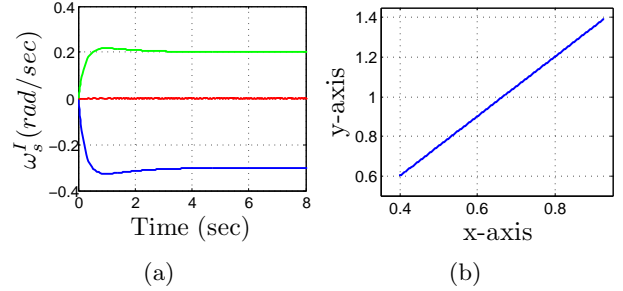


Fig. 9. Angular velocity and phase plane of xy position tracks the line trajectory.

4 Discussion

In conclusion we say that both the control strategies derived using the geometric approach, without parametrization, illustrate a more general philosophy on the control design keeping the mechanical notions of the system. To the best of our knowledge this is the first instance where such a strategy has been employed to a nonholonomic system. The first feedback strategy results in a continuous feedback law which stabilizes the desired orientation exponentially. In the second strategy, using the notions of the affine connection and a transport map on tangent spaces, an error function and a tracking controller is derived. An intermediate results while proving the stability is $\nabla_{\omega_s^s} e_\omega = f_{PD}$, which provides an interpretation about feedforward control f_{FF} . The closed-loop system has the property that $\nabla_{\omega_s^s} e_\omega$

vanishes along the trajectory. That is, if $\dot{e}_x = (\dot{\mathbf{x}} - \dot{\mathbf{x}}_d)$ is zero at initial time, it will remain zero at final time. And if we keep $\omega_d^s = \dot{e}_3$ we get the $\dot{e}_x(0) = \dot{e}_x(T) = 0$ for all time, and the result is contact position and a reduced attitude stabilization.

Appendix

4.1 Proof of Lemma 1:

In this section we will compute the Lie brackets of F_{cl} and g_i , where $i = 1, 2, 3$. Given two vector field $X, Y \in TQ$ the Lie derivative (bracket) of Y along X is

$$[X, Y] \equiv \frac{d}{dt}|_{t=0} \Phi_t^*(Y),$$

where Φ is the flow of X and $\Phi_t^*(Y)$ is pull-back of a vector field Y . From system (18), the Lie bracket of F_{cl} and g_i will be

$$[F_{cl}, g_i](q) = -[g_i, F_{cl}](q) = -\frac{d}{dt}|_{t=0} (D\Phi_t^{g_i}(q))^{-1} \cdot Y(\Phi_t^{g_i}(q)) \quad (29)$$

where $\Phi_t^{g_i}$ is the flow of g_i . The control vector field $g_i(q) = (\mathbf{0}, M^{-1}\hat{e}_i)$ then flow of g_i is given as

$$\Phi_t^{g_i}(q) = \begin{bmatrix} R_s \\ \omega_s^s + tM^{-1}\hat{e}_i \end{bmatrix}$$

and $(D\Phi_t^{g_i}(q))^{-1}$ is the identity map on the manifold Q . From (29),

$$\begin{aligned} [g_1, F_{cl}](q) &= \frac{d}{dt}|_{t=0} (D\Phi_t^{g_1}(q))^{-1} \cdot Y(\Phi_t^{g_1}(q)) \\ &= \frac{d}{dt}|_{t=0} (D\Phi_t^{g_1}(q))^{-1} \cdot \begin{bmatrix} R_s(\omega_s + tM^{-1}\hat{e}_1)^\wedge \\ M^{-1}K_p(\sum_{i=1}^3 R_s^T \hat{e}_i \times R_d \hat{e}_i) \end{bmatrix} \\ &= \begin{bmatrix} R_s(M^{-1}\hat{e}_1)^\wedge \\ \mathbf{0} \end{bmatrix}. \end{aligned}$$

Similarly, $[g_2, F_{cl}](q)$ and $[g_3, F_{cl}](q)$ are calculated and given as

$$[g_2, F_{cl}] = \begin{bmatrix} R_s(M^{-1}\hat{e}_2)^\wedge \\ \mathbf{0} \end{bmatrix} \text{ and } [g_3, F_{cl}] = \begin{bmatrix} R_s(M^{-1}\hat{e}_3)^\wedge \\ \mathbf{0} \end{bmatrix}.$$

The vectors $g_1, g_2, g_3 \in T_q Q$ are linearly independent since $\{\hat{e}_1, \hat{e}_2, \hat{e}_3\}$ are linearly independent. To see the linear independence, we write all the six vectors as

$$\sum_{i=1}^3 (\alpha_i g_i(q) + \beta_i [g_i, F_{cl}](q)) = 0. \quad (30)$$

Now, for these vectors to be linearly independent, all the scalars α_i 's and β_i 's equal to zero. To show this, substituting the values of g_i and $[g_i, F_{cl}]$ in (30) it follows that

$$\alpha_i M^{-1}\hat{e}_i = 0 \quad (31)$$

$$\beta_i R_s(M^{-1}\hat{e}_i)^\wedge = 0. \quad (32)$$

for all i . Since $\{g_1, g_2, g_3\}$ is linearly independent, (31) will hold only when $\alpha_i = 0$. And

$$\{R_s(M^{-1}\hat{e}_1)^\wedge, R_s(M^{-1}\hat{e}_2)^\wedge, R_s(M^{-1}\hat{e}_3)^\wedge\}$$

is linear independent, then $\beta_i = 0$ to satisfy (32). Hence, the set

$$\{g_1, g_2, g_3, [f, g_1], [f, g_2], [f, g_3]\}$$

are linearly independent on $Q = SO(3) \times \mathbb{R}^3$ of dimensional six and spans the tangent space of the configuration space at any configuration. Therefore, the system is locally controllable.

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